[Total No. of Questions: 09] [Total No. of Pages: 02]

Uni. Roll No. ....

Program: B.Tech. (Batch 2018 onward)
Semester:1
Name of Subject: Mathematics I
Subject Code: BSC-103
Paper ID: 15927

Time Allowed: 03 Hours Max. Marks: 60

NOTE:

1) Parts A and B are compulsory
2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
3) Any missing data may be assumed appropriately

Part – A [Marks: 02 each]

Q1.

a) Define Clairaut's equation.

b) For what values of p, does the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge or diverge?

c) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$.

d) Test the convergence or divergence of the improper integral $\int_0^{\infty} e^{-x} dx$.

e) Using Cayley Hamilton Theorem, Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$.

f) Solve the differential equation $\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

Part – B [Marks: 04 each]

Q2. Find the Maclaurin series for $f(x) = \sin x$.

Q3. Solve the differential equation $(3xy^2 - y^3)dx - (2x^2 y - xy^2)dy = 0$.

Q4. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Q5. Solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ by variation of parameter method.

Q6. Use the rank method to test the consistency of the system of equations $2x + 3y + 4z = 11$, $x + 5y + 7z = 15$, $3x + 11y + 13z = 25$.

Q7. Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \dfrac{2n-1}{n(n+1)(n+2)}$.

Part – C   [Marks: 12 each]

Q8. Solve the differential equation $x^2 \dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} + 2y = x\log x$.

OR

(i) Solve $x\dfrac{dy}{dx} + y = x^3 y^6$.

(ii) Solve the equation $3x^4 p^2 - px - y = 0$.

Q9. Diagonalise the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ and obtain its modal matrix.

OR

Discuss the convergence of the series $\sum_{n=1}^{\infty} \dfrac{4.7.10......(3n+1)}{1.2.3......n} x^n$.

****